



Maths is Treemendous!

LEARNING INTENTIONS

(pupils should be taught...)

- Basic trigonometry using tangents (pre-visit and on-site activity)
- How to calculate mean and median (pre-visit and on-site activity)
- The use of a clinometer or protractor for measuring angles of tall objects (on-site activity)
- How to determine the area of a triangle (on-site activity)
- Risk assessments for working in Clouts Wood (pre-visit activity) and based upon the Risk Assessment activity to be found at: www.countrysideaccess.gov.uk
Go to the site and under Countryside Code, click on learning resource centre.

LEARNING OUTCOMES

(pupils will have learnt...)

- To determine the height of trees using basic trigonometry
- To determine the triangular area between three trees using a simple formula
- That mathematics is everywhere and thus relevant!
- That the use of mathematics in the real environment often has limitations and is based on assumptions

KEY WORDS

TANGENT
MEDIAN

ADJACENT
AVERAGE

HYPOTENUSE
CLINOMETER

MEAN

CARE FOR OURSELVES (our health and well-being)

- Pre-visit risk assessment undertaken by learners
- Appropriate clothing

CARE FOR OTHERS (across cultures, distances and generations)

- Provide opportunities for learners to discuss / question the use of this local wood. Who uses it and why?

CARE FOR THE PLANET (both locally and globally)

- Provide opportunities for discussion of the Countryside Code and the wider importance of woods in the landscape. Its role in sustainable communities.
- The impact of deforestation on soil erosion. Provide opportunities for discussion on causes and other consequences of deforestation.



Resources:

- worksheets
- clinometers
- calculators
- measuring tape

ECM Outcomes:

- Be Healthy
- Stay Safe
- Enjoy & Achieve
- Make a Positive Contribution
- Economic Well-Being

Teaching & Learning Activities

INTRODUCTION

- Make the learning intentions and outcomes clear to the learners.
- To re-cap lesson on use of using tangents and calculation of median and mean.
- To set boundaries to behaviour and range in woods and as appropriate to age group.
- Provide map of woods with areas chosen to undertake session as appropriate to age group.

MAIN ACTIVITIES

Learners are provided with worksheets including Figures 1 and 2 and a practical demonstration of the methodology for measuring vertical angles with clinometers/protractors (depending on resources available – Geography or Biology departments often have clinometers).

The learners are sent off in groups with the worksheets and clinometers to determine the heights of 10 trees, the mean and median heights of the trees and the area of land between three of them.

The groups complete the "Assumptions/Errors" questions

PLENARY

Learners to reconvene and asked to reflect on the activity. Learners to provide answers (should all be of the same order of magnitude). Learners to comment on any apparently anomalous results. Discussion of the assumptions/errors.

HOMEWORK

Learners to measure the heights of their own houses, lamp posts and a tree/shrub/fence that can also be measured directly with a tape measure in order to make comparison of results.

ADDITIONAL ACTIVITIES:

Different learners can determine the heights of the same tree. Are the heights the same? If not, why not?

All the internal angles of a triangle must add to 180° . Therefore, using Figure 1, the third angle from the tree top to the learner must be (in the case of Figure 1) $180^\circ - 90^\circ - 35^\circ = 55^\circ$.

Prove this using the three trigonometric functions.

What would the angle of elevation (θ) be if the tree was 100m tall?

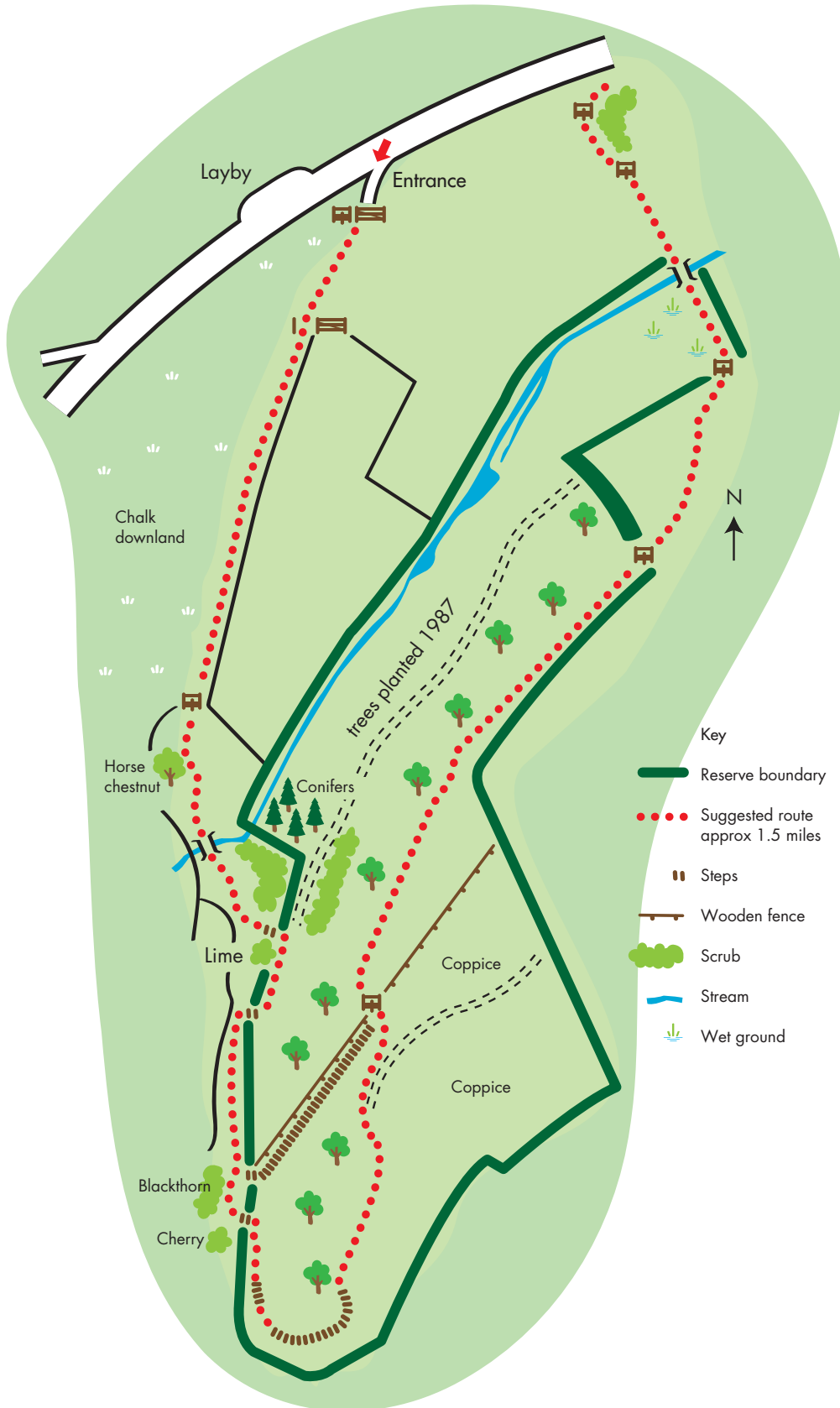
MOST LEARNERS:

Can use the clinometers to determine the angles of the trees, have calculated the heights as well as the areas between three trees and have provided errors/assumptions. Provided one assumption/error for the 'mathematical journey'.

SOME LEARNERS HAVE NOT MADE SO MUCH PROGRESS AND: Use the clinometers to determine the angles and have calculated the area between the trees.

SOME LEARNERS HAVE PROGRESSED FURTHER AND: Have completed the two main exercises, and have provided several well-argued points on the 'mathematical journey' as well as starting the additional activities.

Clouts Wood Nature Reserve Map



CALCULATING THE HEIGHT OF A TREE USING BASIC TRIGONOMETRY

1. The aim of this technique to use the tree to produce a right angled triangle and then use tangent to determine the height. **See Figure 1 on page 6.**
2. Choose a suitable distance from the base of a tree you wish to measure. Measure the distance. This will depend on the height of the tree and you will be able to judge the best distance to measure after a few attempts. Let's say this is **20m**. This is the **adjacent**. Once you have started the procedure you must not move.
3. Measure your height from the floor to your eye level. Let's say this is **1.50m**.
4. Using a **clinometer** measure the angle from your eye to the top of the tree. Let's say this is **35°**. You are looking along the **hypotenuse**.
5. Using your measurements you can now calculate the height of the tree (the **opposite**). By doing the following:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore \tan 35 = \frac{x}{20}$$

$$\therefore 0.700 = \frac{x}{20}$$

By re-arranging the equation we get:

$$0.70 \times 20 = x$$

$$\therefore x = 14.00\text{m}, \text{ so the tree is } 14,00\text{m high...or is it?}$$

Remember, you measured the height from the floor to your eye level, and we said that this was **1.50m**

$$\therefore \text{the tree is } 14.00 + 1.50 = \underline{15.50\text{m}}$$

6. In small groups now select 10 trees that are close to each other and determine their height. What errors may have occurred in the determination of the height of the trees? See if you can think of at least 3.

Once you have done this determine the **mean** and **median** heights of the 10 selected trees.

Are the answers different; if so why? Which do you think is the best measure of 'average' height and why.

Assuming they have not actually grown in the meantime, do you think if you came to measure the height of the trees in winter, you would get the same height for the same trees? If not, why not?

DETERMINE THE AREA OF TRIANGLES USING THREE TREES

1. Choose three trees you have determined the heights of that do not form a right angled triangle to each other (See Figure 2 on page 6). Try to choose trees that have a completely overlapping canopy, i.e. the leaves of each of the three trees merge.
2. Using Figure 2 determine the area of land between the three trees. You will need a measuring tape to do this.

Mathematics

NOW, LET'S GO ON A MATHEMATICAL JOURNEY TO SEE WHAT WOULD HAPPEN TO THE SOIL IF ALL THE TREES IN CLOUTS WOOD WERE CUT DOWN...

A typical rain drop is 2mm in diameter and has a 22.5km/hr velocity...

The maximum diameter of the falling raindrop as it hits the ground is 2mm and the area of a circle is determined by πr^2 ...

Thus each raindrop has an area of 3.142mm² and let's say that each drop erodes three soil particles...

Using the area from Figure 2, the area formed by the triangle between the trees is 25m²...

3.142mm² in m² = 0.00000314m²...

Thus 7 961 783.44 rain drops fall in area of 25m²...

How many soil particles are eroded within the 25m² triangle measured...?

We've said that there are 3 particles in 3.142mm² of soil thus each particle = 1.05mm² ...

With the same third dimension, a cube of this area would be 1.08mm³ per soil particle...

Are you still awake...?

3 particles per raindrop x 7 961 783.44 rain drops in 25m² = 23 885 350.32 particles, with each particle being 1.08mm³...

Thus 23 885 350.32 x 1.08 = 35 796 178.34mm³ eroded within 25m². This = 0.036m³ within 25m²...

Clouts Wood = 13 hectares in size. This = 130 000m²

There are 5200 25m² in 13 hectares thus 5200 x 0.036 m³ = 187.20m³...

In other words 187.20m³ of surface soil particles would be eroded in Clouts Wood if there was a uniform covering of 2mm raindrops.

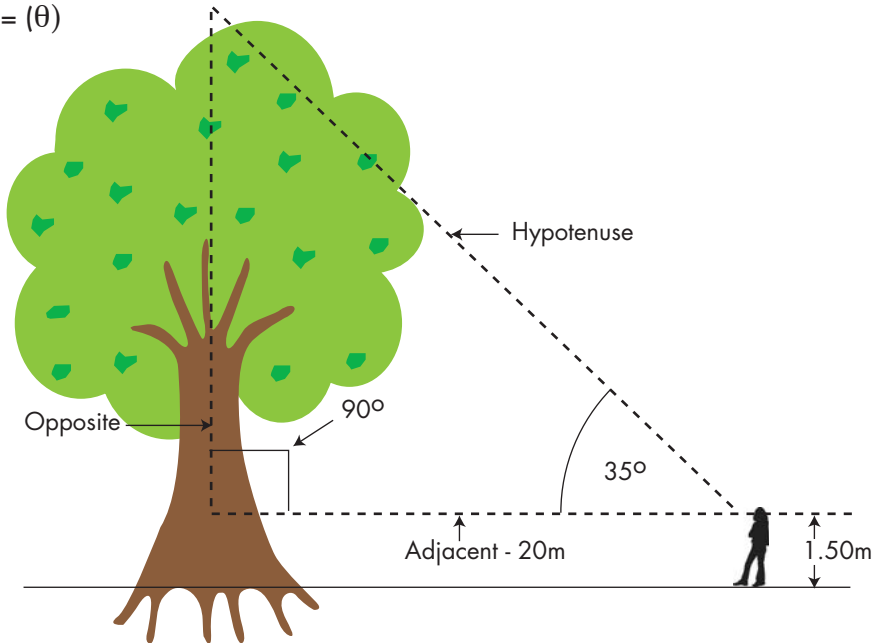
To put this in perspective, the new Mini Cooper has an interior volume of 2.18m³ thus the equivalent of 85.87 Mini Cooper interiors would be eroded by a uniform covering of 2mm raindrops.

The Amazon rainforest has an area of 725 000 000 000m²... I'll let you do the calculations for the amount of erosion caused by deforestation...!!!

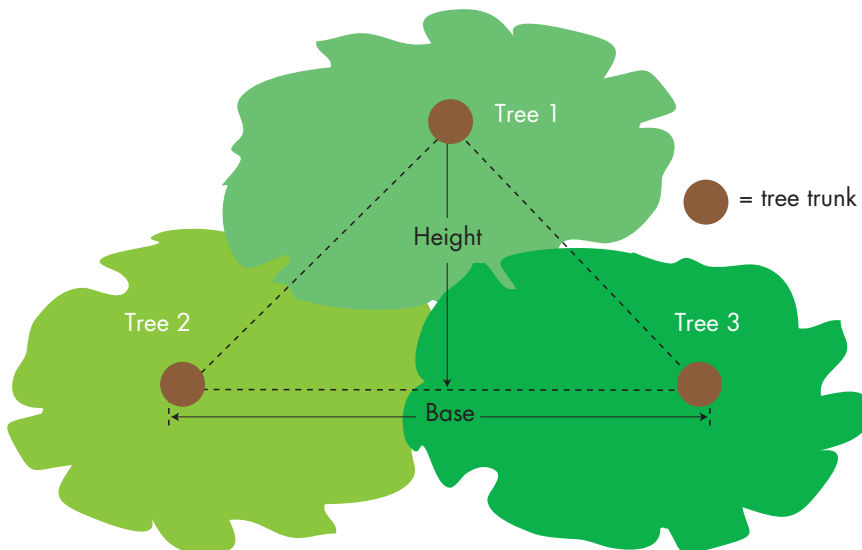
THERE ARE MANY ASSUMPTIONS AND POSSIBLE ERRORS IN THE ABOVE CALCULATIONS. WRITE DOWN THREE:

**FIGURE 1
USING A RIGHT ANGLED TRIANGLE TO DETERMINE THE HEIGHT OF A TREE.**

Angle $35^\circ = (\theta)$



**FIG 2
A BIRD'S EYE VIEW OF THREE TREES CHOSEN**



The formula for calculating the area of a triangle is: $\frac{1}{2} \text{ Base } \times \text{ Height}$

so for example if the distance from Tree 2 to Tree 3 is 10.00m and the distance from Tree 1 to the base line is 5.00m $\therefore \frac{1}{2} (10.00 \times 5.00) = 25\text{m}^2$